MATH 141 HOMEWORK 3 DUE DATE: MONDAY, MAY 22ND

Name: Solutions

This homework is due Monday, May 22nd. If you have questions regarding any of this, feel free to ask during office hours or send me an email. When writing solutions, present your answers clearly and neatly, showing only necessary work.

1. Calculate the derivative of $f(x) = \sin(4x^2 - x) \cdot (3x^2 + 2)$. Product + Chain

$$(8x-1)\cos(4x^2-x)(3x^2+2) + \sin(4x^2-x)\cdot 6x.$$

Answer:

2. Calculate the derivative of $f(x) = \sec(2x) + 5x + \csc(x^2)$.
chain power chain

Answer: $2\sec(2x)\tan(2x) + 5 - 2x\cot(x^2)\csc(x^2)$

3. Find $\frac{dy}{dx}$ if $y^4 + \cos(xy) = \tan(x^2) + e^y$. Simplify you answer.

$$4y^3\frac{dy}{dx} - \left(x\frac{dy}{dx} + y\right)\sin(xy) = 2x\sec^2(x^2) + e^y\frac{dy}{dx}$$

$$(4y^3 - x\sin(xy) - e^y)\frac{dy}{dx} = 2x\sec^2(x^2) + y\sin(xy)$$

$$\frac{dy}{dx} = \frac{2x\sec^2(x^2) + y\sin(xy)}{4y^3 - x\sin(xy) - e^y}$$

Answer:

4. Let $x^3y + xy^3 = 2$.

(a) Find $\frac{dy}{dx}$. Simplify your answer.

$$3x^2y + x^3\frac{dy}{dx} + y^3 + 3xy^2\frac{dy}{dx} = 0$$

$$(x^3+3xy^2)\frac{dy}{dx} = -y^3 - 3x^2y$$

$$\frac{dy}{dx} = \frac{-y(y^2+3x^2)}{x(x^2+3y^2)}.$$

Answer:____________________

(b) Find $\frac{d^2y}{dx^2}$. You do not need to simplify your answer.

$$\frac{d^2y}{dx^2} = \frac{x(x^2+3y^2)\left[-y\left(2y\frac{dy}{dx}+6x\right)-(y^2+3x^2)\frac{dy}{dx}\right] + y(y^2+3x^2)\left[x^2+3y^2+x\left(2x+6y\frac{dy}{dx}\right)\right]}{(x(x^2+3y^2))^2}$$

Answer:____________________

5. Let $x^2 - y^2 = 7$.

(a) Find $\frac{dy}{dx}$

$$2x - 2y\frac{dy}{dx} = 0$$

$$2y\frac{dy}{dx} = 2x \qquad \frac{dy}{dx} = \frac{x}{y}$$

Answer:____________________

(b) Use your answer from part a) to find the equation of tangent line through the point $(4, -3)$. If you did not get an answer for part a) you may assume $\frac{dy}{dx} = 5$ at $(4, -3)$. (Note that this is *not* the correct answer).

$$\left.\frac{dy}{dx}\right|_{\substack{x=4\\y=-3}} = \frac{4}{-3} \qquad y+3 = -\frac{4}{3}(x-4)$$

Answer: $y = -\frac{4}{3}x + \frac{7}{3}$.

The End.